

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. State Cayley – Hamilton theorem and show that it

is satisfied by the matrix $A = \begin{bmatrix} -11 & -10 & 6 \\ 5 & 4 & -5 \\ -20 & -20 & 4 \end{bmatrix}$.

17. What do you mean by symmetric and antisymmetric tensors? Show that any contravariant (or) covariant tensor of second rank can be expressed as the sum of a symmetric and an antisymmetric tensor of the same rank.

18. Solve the differential equation.

$$\frac{d^2y}{dx^2} + 4 \frac{dy}{dx} + 3y = 65 \cos 2x.$$

19. Prove that : $e^{\frac{xz-1}{z^2}} = \sum_{-\infty}^{+\infty} z^n J_n(x)$.

20. Derive Green's function for three dimensional Helmholtz equation.

APRIL/MAY 2024

GPH11/DPH11 — MATHEMATICAL PHYSICS — I

Time : Three hours

Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Show that the vectors $(1, 2, -3)$, $(1, 3, -2)$ and $(2, -1, 5)$ are linearly independent.
2. What is the characteristic equation of a matrix $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$?
3. What do you mean by real and Dummy indices in tensor analysis?
4. If A_{ij} is an antisymmetric tensor, what is the value of A_{11} ? How?
5. Solve : $(1-x^2)(1-y)dx = xy(1+y)dy$.



6. Solve the differential equation $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$.

7. Prove that: $P_{2n+1}(0) = 0$.

8. Give the Rodrigue's formula for Hermite polynomials.

9. State two properties of Delta function.

10. What is Green's function? Give its symmetry properties.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) Show that the vectors $(u+v)$, $(v-u)$ and $(u-2v+w)$ are linearly independent provided (u, v, w) are linearly independent.

Or

(b) Find the eigen values and normalized eigen

vectors of the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$.

12. (a) Prove that the addition of tensors is commutative and associative.

Or

(b) Mention the properties of Levi - Civita symbol, and show that $\sum_{j,k=1}^3 \epsilon_{ijk} \epsilon_{ljk} = 2\delta_{il}$.

13. (a) Solve the differential equation.

$$\frac{d^2y}{dx^2} - y = \frac{2}{1+e^x}.$$

Or

Solve the equation

$$\frac{dy}{dx} + xy = x^3 y^3.$$

14. (a) Show that $P_n(x)$ is the coefficient of z^n in the expansion of $[1 - 2xz + z^2]^{-\frac{1}{2}}$ in ascending powers of z .

Or

(b) Obtain the generating function for Hermite polynomials.

15. (a) Show that $f(a, x) = \frac{1}{a} f(x)$.

Or

(b) Find one dimensional Green's function for the boundary value problem.

$$y''(x) + y(x) = f(x), \quad y(0) = 0 \text{ and } y(1) = 0.$$